Heat Transfer Between Fluid Flow and Fractured Rocks

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ABSTRACT

Heat transfer between fluid flow and fractured rocks is governed by a very complex mechanism. The challenge of modeling this heat transfer process results from the complex geometry of rock fracture networks, variation of rock and fluid properties with temperature, and uncertainty of the flow and heat transfer types. As the heat transfer models are based on the fracture models and fluid flow models, there can be several approaches to model the heat transfer between fluid and fractured rocks. This study starts with a review of the fracture and flow models on which the heat transfer models are based. To clearly present the models, the rock fracture models are grouped into two categories: Fracture Continuum Models (FCM) and Discrete Fracture Models (DFM). A new heat transfer model based on stochastic DFM, GEOFRAC, is then introduced. A thermal drawdown equation for a geothermal reservoir is derived based on the simulation results of the new heat transfer model.

Introduction

Modeling of heat transfer in fractured rocks is of particular importance for energy extraction analysis in engineered geothermal systems (EGS), and therefore represents a critical component of EGS system design and performance evaluation (Hao et al., 2013).

Generally, the development of a heat transfer model starts with a fracture model, followed by a fluid flow model and finally the heat transfer model. Therefore, one needs to define the fracture and flow models before analyzing the heat transfer models. There have been many models built to simulate fractures, for example the Fracture Continuum Model (FCM) (Reeves, 2006), the Equivalent Continuum Model (ECM) (Wu, 1999), the Dual Continuum Model (DCM) (Pruess and Narasimhan, 1985), the Disc-shaped Orthogonal Fracture Model (Long et al., 1985), and the Stochastic Discrete Fracture Model (DFM) (Dershowitz, 1985; Veneziano, 1979). Although the listed fracture models have different features and applications, they can basically be grouped into two categories: Fracture Continuum Models (FCM) and Discrete Fracture Models (DFM).

Correspondingly, the conceptual approaches to model fluid flow in fractured rocks fall into the same two categories: continuum approach and discrete fracture approach. The first approach idealizes the fractured rock mass as a porous medium, when the scale of the rock mass is sufficiently large relative to the fracture geometry. It uses coefficients to parametrize the relevant physical properties of the entire rock mass, for example, $k_{ij}$ for the hydraulic conductivity of the rock mass. The second approach conceptualizes the rock mass as a set of impermeable blocks, separated by systems of fractures that are idealized as networks of planar conduits (Baca & Arnett, 1984). It treats each fracture separately, producing the locations, orientations, dimensions and connections.

After the fracture and flow models are set up, the heat transfer models can be built accordingly. For the FCM, because the rock mass with fractures is treated as a continuum, and fluid is treated as another continuum within the rock, the heat transfer process is governed by Fourier’s law of heat conduction (Hao et al., 2013). One thermal conductivity coefficient is used to take into account both the rock and fluid heat conductivity. Conversely, the DFM does not abstract the details of the fluid flowing in each rock conduit, so the heat transfer between fluid flow and rock is mainly governed by explicit heat convection equations. A proper heat convection coefficient should be chosen to represent the dimensions and flow state in each fracture.

To better understand the difference between the two heat transfer approaches, it is necessary to start with comparing fracture and flow models. In this paper, typical FCM and DFM examples are chosen as the basis of the fluid flow and heat transfer models, and the models are compared. Then, a new approach to heat transfer based on a DFM, GEOFRAC, is introduced. GEOFRAC is used to generate a stochastic discrete fracture network. GEOFRAC-FLOW is used to calculate the flow. On the basis of the two, the heat transfer problem is simplified as fluid flow between two paralle
Fracture Continuum Model

In the world of mechanics, a continuum is defined as a volume of material whose properties can be represented by one point or one infinitesimal element. The discrete behavior of individual fractures is represented by an average property over a representative elementary volume (REV). An example of a FCM developed by Reeves (2006) is presented in the following.

\[ T = \begin{bmatrix} n_2^2 + n_3^2 & -n_1n_2 & -n_1n_3 \\ -n_1n_2 & n_1^2 + n_3^2 & -n_2n_3 \\ -n_1n_3 & -n_2n_3 & n_1^2 + n_2^2 \end{bmatrix} \] (2)

where,

- \( n_1 = \cos \alpha \cos \omega \)
- \( n_2 = \cos \alpha \sin \omega \)
- \( n_3 = -\sin \alpha \)

\( \alpha \) is the fracture plunge (90°-dip);
\( \omega \) is the fracture trend (strike-90°).

For multiple fracture sets, the hydraulic conductivity tensor is the sum of the hydraulic conductivity tensors of all the individual fracture sets:

\[ k_{ij} = \sum_{m=1}^{N} k_{ij}^m \] (3)

where,

- \( N \) is the number of fracture sets
- \( k_{ij}^m \) is the hydraulic conductivity tensor

Mass and Heat Transfer Based on the FCM

After the permeability tensor is built for the rock mass in the specific direction we need, the rock mass is treated as a heterogeneous anisotropic porous medium. This is a simplification that makes the process very convenient for modeling. In the field of environmental engineering dealing with soil contamination, petroleum engineering dealing with oil extraction, or hydraulic engineering dealing with groundwater movement, similar processes are modeled. Existing software and model codes using finite element or finite difference methods for numerical solutions can be used.

In the model developed by Reeves (2006), the Finite Element Heat and Mass Transfer code (FEHM) developed by Zyvoloski et al. (1988) is used. This code was written to simulate 3-D mass and heat transfer in porous rock. It considers two fluid phases, gas and liquid. The mass transfer is governed by Darcy’s law, and heat transfer is governed by Fourier’s law of heat conduction (Zyvoloski et al., 1988).

Summary of the FCM

The FCM is based on the concept that the fractured rock can be treated as a continuum when the volume of the rock is large enough for this approximation. The FCM is based on the geometry of the fracture sets, so field data like fracture strike, dip, aperture, and spacing, can be employed to build the fracture permeability tensor. The hydraulic conductivity of the FCM can reflect the anisotropy of hydraulic conductivity in the fractured rock. The FCM heat transfer model is quite simple and computationally efficient. It is often used to simulate large volume geothermal reservoir problems.

However, such a model does not consider the real fracture geometry; also the assumption of a continuum often loses too much important information about the rock fractures. When valid ap-
proximations cannot be made for a geothermal reservoir, discrete fracture models may offer more accurate solutions.

Discrete Fracture Model

Geometric and mechanical characterization of rock fractures is the basis for most of the work of engineering geologists, civil and mining engineers when dealing with rock masses (Dershowitz and Einstein, 1988). However, the complete description of fractures is difficult because of their three-dimensional nature and the limited information we can get from outcrops, borings, tunnels, tracer logs, and micro seismic events. Researchers from Lawrence Livermore National Laboratory, depicted a realistic fracture network based on their observation of fractures in rock masses, as shown in Figure 2 (Settgast et al., 2012).

Figure 2. A realistic fracture network derived from experimental data (Settgast et al., 2012).

Also, the analysis of micro seismic events data can be used to depict the fracture networks near an exploration borehole. Figure 3 shows a picture of fracture network near the Habanero-1 exploration well in the Cooper Basin, Australia (Barton et al., 2013).

Figure 3. Network of fractures local to Habanero 1 viewed from the South-east. Color indicates proximity of the fractures to frictional failure (Barton et al., 2013).

Modern technology can only provide limited information on the fracture network. To reproduce the fracture network in the model, one needs to rely on stochastic models. To provide parameters for this stochastic approach, statistical procedures have to be used to characterize the fracture orientations, shape, size, spacing and intensity. Researchers have proposed a number of statistic distributions for the fracture geometric characteristics.

With the help of statistical studies on the fractures, stochastic discrete fracture models were developed, e.g. Veneziano (1978), Dershowitz (1985), Ivanova (1995). This modeling approach has been applied in software such as Fracman and GEOFRAC, which can operate on different platforms. The following is a brief summary of the new GEOFRAC model using MATLAB.

GEOFRAC is a three-dimensional (3D), geology-based, geometric-mechanical, hierarchical, stochastic model of natural rock fracture systems (Ivanova et al., 2012). The model represents fracture systems as 3D networks of intersecting polygons, generated through spatial geometric algorithms that mimic the mechanical processes of rock fracturing in nature. The fracture generation process is visualized in Figure 4.

In GEOFRAC, stochastic processes are used to decide on fracture size, aperture, tessellation, and rotation. Detailed information can be found in Ivanova et al. (2012). With the help of built-in probability functions in MATLAB, these stochastic processes can be implemented easily. Geometric information on all the fractures is produced by the model. Validation of this fracture model was proven by Sousa et al. (2012), who analyzed the connectivity of fracture networks.

Flow Model Based on DFM

On the basis of GEOFRAC, a flow model was developed by Sousa et al. (2012). Because this discrete fracture model accounts for individual fractures, fluid flow in the fractures can be captured by analyzing the flow in each fracture. The fluid flow through fractures is assumed to be governed by Poiseuille’s law, also known as the cubic law (Witherspoon et al., 1980). The cubic law is an analytical solution for laminar flow between smooth parallel plates. Figure 5 is a sketch of flow between parallel plates in the x direction.

\[
Q = \frac{w \delta^3 \Delta P}{12 \mu \Delta L} \tag{4}
\]

where,
$Q$ is volumetric flow rate;
$w$ is fracture width;
$\mu$ is fluid dynamic viscosity;
$\delta$ is aperture between the plates;
$\Delta P/\Delta L$ is pressure gradient.

This equation is an idealized case. In reality, flow through fractures differs from the cubic law because of the following four factors:

1. Surface roughness of a fracture.
2. Flow path tortuosity. The path in the fracture is not straight; the direction changes cause water head loss.
3. Surface contact in a rock fracture. The aperture of a fracture is not constant; there are regions where the two fracture surfaces contact each other.
4. Local water head loss at fracture intersections.

Witherspoon et al. (1980) modified Equation (4) to account for roughness of the fracture surfaces to the form of

$$Q = \frac{w\delta^3 \Delta P}{12f \mu \Delta L} = \frac{w\delta^3 \Delta h \gamma}{12f \mu \Delta L}$$

$$f = 1 + 6\left(\frac{\varepsilon}{\delta}\right)^{1.5}$$

where,

$f$ is a reduction factor (Witherspoon et al., 1980)
$\varepsilon$ is the surface roughness;
$\Delta h/\Delta L$ is fluid head loss;
$\gamma$ is the unit weight of the fluid.

Refer to Equation (4) for other parameters

This modified form has been validated by Witherspoon et al. (1980) in their experiments, and is used in GEOFRAC-FLOW to govern the fluid flow in fractures.

For flow in fracture networks, mass and energy conservation equations are used as governing equations. The fluid head loss along a fracture can be calculated with:

$$\Delta h = \frac{f 12\mu \Delta L}{\gamma \delta^3} Q$$

As shown in equation (7), head loss is proportional to the length of the fracture, reciprocal to the width and the aperture cubed. Aperture is treated as constant; width is stochastically distributed, so the length of the flow path determines the head loss in of the flow. Fluids tend to flow in the direction which has the greatest pressure gradient, and travels along the paths causing smallest head loss. In the model, paths are chosen as the shortest connection between the inlet fracture and outlet fracture. These connections can be found by using the Dijkstra’s algorithm (Dijkstra, 1959). After the paths are found, a flow path network is built as shown in Figure 6.

To reduce computational cost, the fracture network is simplified by using one branch to equivalently represent the fractures from one node (intersection) to another. The equivalent fracture aperture, length, and width are expressed below.

$$\delta_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{\delta_i}}$$

$$l_{eq} = \frac{1}{\sum_{i=1}^{n} l_i}$$

$$w_{eq} = \frac{\sum_{i=1}^{n} w_i l_i}{\sum_{i=1}^{n} l_i}$$

where,

$l_i$ and $\delta_i$ as the length and aperture of the $i^{th}$ fracture;
$l$ is the total length of the series of fractures.

The mass and energy conservation equations are solved at the nodes in the network to calculate the flow rate in all the branches. Validation of the flow model has been performed with parametric studies (Vecchiarelli et al., 2013).

**Heat Transfer Model Based on DFM:**

Based on the flow model, a heat transfer model was developed by Yost and Einstein (2013). A sketch of this model is shown in Figure 6. The model has the following assumptions:
1. Rock temperature is uniform and constant.
2. Fractures are considered to be parallel plates.
3. Flow in the fracture is laminar and at steady state.

\[
T_2 = T_r - (T_r - T_1) \exp \left( -\frac{h_TPL}{mC_p} \right)
\]

\[
h = \frac{Nu \times k_f}{D_h}
\]

where
- \(T_2\) is the inlet temperature;
- \(T_1\) is the outlet temperature;
- \(T_r\) is rock temperature;
- \(h_T\) is heat convection coefficient;
- \(P\) is perimeter \(2(\delta + w)\);
- \(L\) is the fracture length;
- \(\dot{m}\) is the mass flow rate;
- \(C_p\) is the specific heat capacity.

\(k_f\) is the fluid heat conductivity

\(D_h\) is the hydraulic diameter of the conduct

\(Nu\) is Nusselt number, which is the ratio of convection to pure conduction heat transfer:

\[
Nu = \frac{h_f}{k_f}
\]

where, \(L_C\) is the characteristic length, it is the hydraulic diameter in equation (12).

\(Nu\) is related to flow state, and the dimensions of the parallel plates. Laminar flow between two isothermal parallel plates, usually have long thermal entrance length \(L_{eh}\). \(L_{eh}\) is defined as the distance required for the Nusselt number to decrease to within 5% of its fully developed value \(Nu_{\infty}\).

\[
L_{eh} = 0.017Re_DPr
\]

where
- \(Re_D\) is the Reynolds Number, for laminar flow, \(Re_D < 2300\);
- \(Pr\) is Prandtl number, \(Pr = \frac{C_p \mu}{k} = \frac{4200 \times 0.001}{0.58} = 7.24\);
- \(D\) is the hydraulic diameter at the magnitude of 0.01m.

From the above analysis, we can see that the thermal entrance length \(L_{eh}\) is approximately 3m, which is longer than \(L\), so we use average Nusselt number for flow between isothermal parallel plates of the length \(L\) (Mills, 1995).

\[
Nu_{D_h} = 7.54 + \frac{0.03(D_h/L)Re_{D_h}Pr}{1 + 0.016[(D_h/L)Re_{D_h}Pr]^{2/3}}
\]

\[
Re_{D_h} \leq 2800
\]

This model explicitly calculates the heat transfer in each fracture and obtains the temperature of each node by weight averaging. Although the assumptions idealize the physical process, the model can capture the basic heat transfer problem between the rock and the fluid. Figure 8 is an example of the results produced by the three coupled models. The size of the reservoir is 10*10*10 m³, the blue colored values indicate the flow rate (L/s) produced by the flow model. The purple values are the temperatures of the nodes (intersection of fractures). The maximum Reynolds number of all the flows in the 8 fractures is 1086.24, satisfying the laminar flow assumption very well. The resulting temperatures are reasonable: because the small aperture and roughness of the fractures make the heat transfer between the rock and fluid very efficient, the fluid reaches thermal equilibrium in the rock before it reaches the outlet point of the reservoir.

**Figure 8. Example of the results produced by the heat transfer model.**

**Derivation of Thermal Drawdown Equation**

Service life is one of the key factors determining the feasibility of an EGS. Researchers and practitioners are interested in how the temperature in an EGS draws down over time — faster cooling means more frequent re-drilling and re-stimulation, and lower economic attractiveness. With the conclusion that the temperature of the water in the production well reaches the same temperature as the surrounding rock before leaving the reservoir, a very simple thermal drawdown equation is derived that treats the reservoir using a lumped thermal capacitance model. This equation can quickly give the decision makers an estimation of the service life of the EGS.

The assumptions for the equations are:
1. The temperature of the water at the outlet is the same as the average rock temperature.
2. There is no water leakage in the reservoir.
3. The volume of the thermal reservoir is defined as the volume such that there is no heat transfer at the boundary of the reservoir.

A figure of the geothermal reservoir with related parameters is shown in Figure 9.

The equations are derived as follows:

Energy conservation equation:

\[
\Delta T_R \rho_R C_{PR} = -(T_R - T_0)Q \Delta t \rho_R C_{PW} + Q_0 S_R \Delta t
\]

\[
\frac{dT_R}{dt} \rho_R C_{PR} = -(T_R - T_0)Q \rho_R C_{PW} + Q_0 S_R
\]

Initial Condition: \(T |_{t=0} = T_{R0}\)
where,

\( T_w \) is the temperature of injection water;

\( T_R \) is the temperature of rock;

\( Q \) is the total flow rate of water;

\( Q_0 \) is the tectonic heat flow;

\( V_R \) is the volume of the geothermal reservoir;

\( \rho_R \) is rock density;

\( \rho_W \) is water density;

\( C_{PW} \) is the specific heat capacity of water;

\( C_{PR} \) is the specific heat capacity of rock;

\( S_R \) is area of the geothermal reservoir.

The first term in equation (17) is the heat produced by the rock, the second term is the heat carried away by the fluid, and the third term is the subsurface heat flow into the reservoir.

Solution to equation (18):

\[ T_R = D_3 e^{-D_3 t} + \frac{D_2}{D_1} \quad (19) \]

\[ D_1 = \frac{Q \rho_W C_{PW}}{V_R \rho_R C_{PR}} \quad (20) \]

\[ D_2 = \frac{Q_0 S_R + T_w Q \rho_W C_{PW}}{V_R \rho_R C_{PR}} \quad (21) \]

\[ D_3 = T_{R0} - T_w - \frac{Q_0 S_R}{Q \rho_W C_{PW}} \quad (22) \]

From equation (19), we can see that, the thermal drawdown of the geothermal reservoir follows an exponential curve, as shown in Figure 10. If the flow rate is constant, the heat extracted from a geothermal reservoir will decrease along an exponential curve.

A parametric study is conducted to analyze the sensitivity of the thermal drawdown equation. Figure 10 shows the effect flow rate has on the thermal drawdown of the reservoir. The size of the reservoir is 1000m*500m*500m. With higher injection rate, the reservoir cools down faster.

From the thermal drawdown equation, we can also see that the drawdown speed is also sensitive to the size of reservoir, as shown in Figure 11. In this plot, the flow rate is 0.01 m³/s. The drawdown time is very long for large thermal reservoirs.

Reservoir size and flow rate are the two main factors that determine the service life of a geothermal reservoir. The smaller the injection rate and the larger the reservoir, the longer the service life. The equation also implies that the effect of subsurface heat flow is negligible for most practical reservoirs.

The thermal drawdown equation derived from this lumped thermal capacitance model provides a very simple and quick approach to evaluate the service life of a geothermal reservoir. However, there are two factors to consider when thinking about how the results of this simplified model will deviate from reality. The first is that when...
a reservoir is cooling down, the rock near the production well tends to have higher temperature than that in the injection well. Similarly, the rock located near the flow paths will be cooler than rock located far away from the flow paths. The deviation between our simplified approach and a more complex approach depends largely on the conductivity of the rock relative to the rate of energy extraction from the reservoir and the average distance between the flow paths. Highly conductive rock, with a low rate of removal, and with short distances between the flow paths will see a relatively uniform temperature in the reservoir and results consistent with the lumped thermal capacitance model, while less conductive rock with high energy extraction rate and long distances between the flow paths are more likely to deviate in their results.

**Conclusion**

Currently, the DFM has advantages over other models because it can simulate the geometry of real fractures. It is based on statistical features summarized from field data, which will make the stochastic processes used in the model reliable. Also, the geometric information provided for every single fracture makes it possible to analyze the physical processes in each fracture. All these advantages will result in a good accuracy of the simulation. However, the disadvantage of DFM is that this approach is computationally limited for large reservoirs because of the memory space the model occupied to save geometry information. The solution can either be to use super computers or to optimize the algorithm.

The fluid flow and heat transfer models based on GEOFRAC capture the physical processes in the fractures explicitly. The energy balance algorithm to calculate flow is widely used and proved to be reliable when considering related fields such as groundwater hydrology. The simplifications and assumptions of the flow model may lead to inaccurate results. Also, there is still controversy on the definition of hydraulic aperture and the reduction factor in the cubic law. The assumption of uniform rock temperature makes the heat transfer model efficient and reasonably accurate. The thermal drawdown equation is very efficient for roughly estimating the operation time of a thermal reservoir. However, the assumptions such as uniform rock temperature distribution may not be appropriate and may cause inaccuracy in the results.

Overall the coupled fracture geometry—fracture fluid flow—heat transfer models based on stochastic DFM, GEOFRAC, produce satisfactory results. One has to be aware, however, that several simplifications are made.

**References**


