ABSTRACT

The key to attaining EGS electric power generation is controlling fracture stimulation in low-permeability heat exchange volumes between input and output wellbores. Too few stimulation fractures means too little flow of heat-recharged water between input and output wellbores. Too extensive fracture stimulation could mean significant channeled flow in the heat exchange volume leading to local cooling and lost heat exchange capability.

Controlling EGS fracture stimulation requires understanding:

i. How/why in situ fractures are distributed;

ii. How fracture connectivity controls in situ fluid flow;

iii. How raising heat-exchange volume fluid pressure can induce greater fracture connectivity.

We understand from the systematics of well-log power-spectra that in situ fracture distributions are characterized by long-range spatially-correlated grain-scale-density fluctuation ‘noise’ that scales inversely with spatial frequency, \( S(k) \sim 1/k \), over five decades, \( -1/km < k < -1/cm \). We understand from well-core poroperm fluctuation systematics that spatial changes in porosity \( \phi \) control spatial changes in permeability \( \kappa \) as \( \delta \phi_j \sim \delta \log \kappa_j \), \( j = 1 \ldots n \), where \( \delta \phi \) and \( \delta \log \kappa \) are respectively \( n \)-valued zero-mean/unit-variance fluctuation sequences of well-core porosity and log(permeability) data. While we do not understand in any controlling detail how raising in situ fluid pressure in a crustal volume can be made to induce fracture connectivity in the volume, we note here that integrating the poroperm fluctuation relations empirically describes fracture connectivity as \( \kappa \sim \kappa_0 \exp(\alpha(\phi-\phi_0)) \) in terms of the ratio of standard deviations from sample means \( \alpha \equiv \sigma(\log \kappa)/\sigma(\phi) \) that reflects the degree to which in situ grain-scale fractures are connected in fracture-networks. Small values of \( \alpha \) correspond to diffuse grain-scale fracture connectivity associated with normally distributed permeability populations. Large values of \( \alpha \) correspond to grain-scale fracture connectivity associated with long-tailed permeability population distributions commonly parameterized as lognormal statistical distributions. The degree of fracture-connectivity in a crustal volume thus appears to be the physical explanation lying behind the lognormal statistical descriptions of well-core permeability distributions. (The physics of fracture connectivity may also lie behind the lognormal statistical descriptions often used to characterise trace element and ore-body population distributions.)

Expression \( \kappa \sim \kappa_0 \exp(\alpha(\phi-\phi_0)) \) implies that for a fixed porosity distribution \( \phi \), increased permeability is associated with increased fracture connectivity in the population of grain-scale fractures. Flow simulations in poroperm media with porosity fluctuations having inverse spectral scaling \( \tilde{S}(k) \sim 1/k \) and permeability fluctuations obeying \( \delta \phi \sim \delta \log \kappa \) illustrate the increased permeability due to increased fracture connectivity. We thus argue that inducing finite shear strain in a crustal volume introduces new grain-scale defects in association with existing porosity and hence creates greater permeability through greater fracture connectivity within the volume. Naturally occurring finite-strain-injection processes would generate a range of values of \( \alpha \) testified to by the range of observed normal-to-lognormal distributions for well-core permeability, trace element abundance, and ore body grades. We infer that permeability enhancement for EGS heat-exchange volumes can proceed along similar lines.

Background

A tendency for oil-field and hydrogeologic well-core permeability populations to be ‘long-tailed’ or ‘lognormally’ distributed has long been noted (Law 1944; Bennion & Griffiths 1966; Freeze 1975; Dagan 1989; Ingebritsen, Ward & Neuzil 2006). While the ‘lognormality’ of permeability distribution has often been disputed in favour of alternative mathematical descriptors (Jensen, Hinkley & Lake 1987; Li & Lake 1995; Ricciardi, Pinder & Belitz 2005), there have been few reported attempts to understand the physics underlying long-tailed permeability distributions. A similar comment can be made on the observed range of normal-to-lognormal distributions for well-core permeability, trace element abundance, and ore body grades. We infer that permeability enhancement for EGS heat-exchange volumes can proceed along similar lines.
Without understanding the physical fundamentals of in situ permeability, there is little likelihood of understanding how to create the higher permeability between wellbores pairs needed to achieve EGS electrical power production. Standard massive hydrofracture operations can induce high flow between country rock and a wellbore (e.g., Mooney 2011) but past EGS projects have conspicuously failed at the far more delicate task of producing significant flow between wellbore pairs (Tester et al 2006; Leary, Malin & Pogacnik 2012).

There exists, however, a great deal of well-log and well-core data that attests to a clear empirical approach to in situ permeability and permeability enhancement. We use these data to formulate a physical approach to the fracture-based flow stimulation in EGS volumes. Our approach focuses on an EGS volume enclosing two or more parallel horizontal wellbores of length \( \ell \) offset by distances \( 2a \ll \ell \) as sketched in Fig 1 (Leary, Malin & Pogacnik 2012).

The Fig 1 EGS conceptual model derives from three constraints on in situ flow velocity:

i. The flow system is not lossy; i.e., injection well fluids entering the EGS heat exchange volume exit the volume via production wells;

ii. At the wellbore radius, fluids flow in/out the wellbore at \( v \sim 0.1 \text{mm/s} \);

iii. At distance \( a \) from the wellbore, fluid velocity is \( v \sim 10 \text{mm/s} \).

Wellbore radius flow \( v \sim 0.1 \text{mm/s} \) yields liters/second wellbore flow for a long (100s of meters) reach wellbore. Remote flow \( v \sim 10 \text{mm/s} \) removes heat from the inter-well heat exchange volume at a rate that can be indefinitely replenished by thermal conduction (the ratio of heat advection to heat conduction – Peclet number \( P_e = c_w \rho_w v/\ell K \) for \( c_w, \rho_w \) heat capacity and density of water, and \( K \) the thermal conductivity of rock – is \( \sim \) unity for water moving over distances \( \ell \sim 100 \text{m} \) at flow rate \( v \sim 10 \text{mm/s} \); cf Ingebritsen et al. 2006; Leary et al. 2012). The three velocity constraints we posit for the EGS volume well-separation \( 2a \ll \ell \) are compatible with a fourth EGS constraint, that the means exist to enhance permeability in the heat-exchange volume. For EGS wellbore too far apart, it is difficult to envision how wellbore-to-wellbore enhanced fracture-based permeability fabric can be generated in a controlled fashion through pressurisation of wellbore fluids.

### Three ‘Rules’ of in Situ Fluid Flow

Inter-wellbore fluid flow mechanics is defined by three wellbore-based empirical ‘rules’. Well-log data establish a ‘universal’ inverse power-law scaling ‘rule’ for the physical-variable fluctuation Fourier transform power \( S(k) \) at spatial frequency \( k \),

\[
\mathcal{R}_1: \quad S(k) \approx 1/k, \quad 1/\text{km} < k < ~1/\text{cm}.
\]

\( \mathcal{R}_1 \) is observed for a wide range geological settings and geophysical variables, and for both horizontal and vertical wells. The scale range for which \( \mathcal{R}_1 \) applies runs from 1cm wellbore sampling of microresistivity logs to multiple-km sampling of spatial fluctuations in sonic velocity, neutron porosity, mass density, gamma activity and electrical resistivity (Leary 2002).

Well-core data from oil/gas field clastic reservoir formations establish a robust relation between zero-mean/unit-variance fluctuations in well-core porosity \( \phi \) and the logarithm of well-core permeability \( \kappa \),

\[
\mathcal{R}_2: \quad \delta \phi \approx \delta \log(\kappa).
\]

\( \mathcal{R}_1 \) can be interpreted to mean that in situ fracture systems are spatially-correlated grain-scale-fracture-density network structures through which in situ fluids percolate on all scale lengths (Leary 2002). \( \mathcal{R} \) can be interpreted that porosity controls permeability through interconnectivity of grain-scale fracture networks that allows fluids to percolate through fracture networks on all scale lengths (Leary & Walter 2008).

Writing \( \mathcal{R}_2 \) explicitly as a relation between zero-mean/unit-variance sequences,

\[
(\phi - \langle \phi \rangle)/\sigma(\phi) \approx (\log(\kappa) - \langle \log(\kappa) \rangle)/\sigma(\log(\kappa)), \quad i=1, \ldots, n,
\]

and integrating, we get \( \log(\kappa) \approx \log(\kappa_0) = \alpha (\phi - \phi_0) \), \( \alpha \equiv \sigma(\log(\kappa))/\sigma(\phi) \) the ratio of standard deviations from the mean of well-core sequences \( \log(\kappa) \) and \( \phi \) respectively, with \( \phi_0 \) and \( \kappa_0 \) corresponding to the minimum values of the porosity and log(permeability) sequences, \( \log(\kappa_0) = \min(\log(\kappa)) \). Exponentiating this expression gives in situ permeability as an empirical function of in situ porosity,

\[
\mathcal{R}_3: \quad \kappa \approx \kappa_0 \exp(\alpha(\phi - \phi_0)),
\]

controlled by empirical parameter \( \alpha \).

\( \mathcal{R}_3 \) closely resembles a formal definition of a lognormal distribution \( \chi, \chi = \exp(\mu+\sigma n) \), for normally distributed zero-mean/unit-variance random number sequence \( n \) with \( \mu \) and \( \sigma \) respectively the median and standard deviation of \( \log(\chi) \). As it is widely understood that lognormal variables as a class of physical phenomena can be understood as the outcome of a multiplicative process of otherwise independent events (e.g., Limpert, Stahel & Abbt 2001), we can naturally construe \( \mathcal{R}_3 \) as arising from varying degrees of connectedness between adjacent grain-scale fracture defects or larger volumes to generate percolating fluid pathways at all scale lengths in crustal rock.
The empirical bases for $R_1$ and $R_2$ having been presented and discussed elsewhere (e.g., Leary, Malin & Pogacnik 2012; Leary & Walter 2008), we here focus on the empirical basis for $R_3$.

### $R_3$ Empirics

The empirical basis for $R_3$ centers on the frequency distributions of well-core permeability. As a general rule, histogram data on well-core permeability show that some permeability distributions resemble normal distribution (‘bell-shaped curve’), while most resemble long-tailed lognormal distributions in which many low-permeability samples are accompanied by a few high-permeability samples. $R_3$ encompasses this permeability phenomenology as a function of parameter $\alpha$. As $\alpha$ increases, the range of permeabilities increases thus generating the long-tail distributions commonly associated with lognormal distributions.

Fig 2 exhibits the $R_3$ range of statistical distributions in permeability $\kappa$ for a fixed normally distributed sequence of porosity 0.1 < $\phi$ < 0.35 and a sequence of values of parameter $\alpha$ from 2.5 to 30. For a fixed (normally distributed) porosity sequence $\phi$, small values of $\alpha$ produce normally-distributed permeabilities (small argument approximation to exponent function), which then migrate to long-tailed $\kappa$-distributions as the maximum permeability increases with increasing $\alpha$. While the long-tailed distributions resemble lognormal distributions, they take their shape for an understandable physical reason rather than as a function of arbitrary statistical parameters governing lognormal distributions.

The $R_3$ synthetic realization of permeability distributions in Fig 2 correspond in range and values to those seen in well-core field data as illustrated in Figs 3-8. Comparison between field data and $R_3$ synthetics is inexact because large-scale changes in the poroperm properties within the reservoir change the range of porosity and permeability of a well-core sequence in ways outside the assumptions of $R_3$. Field data of Figs 3-8 confirm, however, the basic Fig 2 values and the statistical distribution trend associated with parameter $\alpha$ in $R_3$.

In particular, as well-core volume sample permeability increases with increasing $\alpha$, it is logical to associate the value of $\alpha$ with the degree of fracture-connectedness within the volume sampled by a well-core sequence. The value of $\alpha$, and therefore an estimate of fracture connectivity for a given volume of crustal sampled by a well-core sequence, is determined by plotting the logarithm of permeability for a well-core sequence against the porosity of that sequence. Figs 3 and 5 show permeability distributions for sample well-core sequences from a North Sea gas field, and for a tight gas formation in South Australia. The frequency plots are arranged with increasing values of $\alpha$, and thus show the transition from ‘near-normal’ (upper left) to increasingly

![Figure 2](image-url)

**Figure 2.** Population distributions for $\kappa = \kappa_0 \exp(\alpha(\phi - \phi_0))$ for $\alpha = 2.5$ to 30 for a fixed normal distribution $\phi$. As $\alpha$ increases, maximum permeability increases while the minimum permeability remain fixed, hence the frequency distribution become more ‘long-tailed’ (more like a lognormal distribution). A similar sequence characterizes the formal expression of lognormal distributions, $\kappa = \exp(\mu + \sigma n)$.

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![Figure 3](image-url)

**Figure 3.** Population distributions of permeability for eight well-core sequences from a North Sea gas field clastic reservoir formation. Parameter $\alpha$ is determined by plotting well-core porosity against the logarithm of permeability as shown in Fig 4. In some similitude with the synthetic poroperm data of Fig 2, lower values of $\alpha$ are associated with more normal distributions and higher values of $\alpha$ are associated with longer-tailed distributions. Unlike Fig 2, however, the minimum and maximum permeabilities change within the formation so comparison with Fig 2 and $R_3$ is inexact.

![Figure 4](image-url)

**Figure 4.** Parameter $\alpha$ determination by plotting well-core porosity against the logarithm of permeability for well-core poroperm sequences from a North Sea gas field clastic reservoir formation. The $\alpha$ values are given above each plot.
Figure 5. Population distributions of permeability for nine well-core sequences from a South Australia tight gas field clastic reservoir formation. Parameter $\alpha$ is determined by plotting well-core porosity against the logarithm of permeability as shown in Fig 6. In some similitude with the synthetic poroperm data of Fig 2, lower values of $\alpha$ are associated with more normal distributions and higher values of $\alpha$ associated with longer-tailed distributions. Unlike Fig 2, however, the minimum and maximum permeabilities change within the formation so comparison with Fig 2 and R3 is inexact.

Figure 6. Parameter $\alpha$ determination by plotting well-core porosity against the logarithm of permeability for well-core poroperm sequences from a South Australia tight gas field clastic reservoir formations. The $\alpha$ values are given above each plot.

lognormal distributions (lower right). The values of $\alpha$ are determined by the log(permeability)-porosity cross-plots shown in matching positions in Figs 4 and 6 each of the well-core sequences shown in Figs 3 and 5. The range of each cross-plot axis in Figs 4 and 6 is seen in the associated frequency plots in Figs 3 and 5.

Figs 7-8 illustrate $\mathcal{R}3$ on a well-core sample by sample basis by pair-wise comparing the observed sample permeability distribution (left) with the $\mathcal{R}3$-predicted permeability distribution computed through the sample porosity (right). Fig 7 compares North Sea gas field poroperm data for 12 wells in the same clastic formation. Fig 8 compares South Australia tight gas formation well-core data for several formations. The observed sample permeability distributions (pair-left) clearly resemble in distribution shape and numerical range the permeability distributions predicted by $\mathcal{R}3$ sample porosity and $\alpha$ (pair-right). We thus can infer from these two sample poroperm fields that the $\mathcal{R}1$-$\mathcal{R}3$ well-log fracture-fluctuation and well-core fracture-connectivity empirics represent a physically valid approach to reservoir flow systematics.

It is interesting to note that the values of $\alpha$ for the permeable North Sea gas field formations (Figs 3 and 5) are

![Figure 7](image7.png)

![Figure 6](image6.png)
substantially smaller than the values of $\alpha$ for the impermeable South Australia tight gas formations. The logical inference is that the more porous North Sea gas field formations have higher ambient pore-pore connectivity requiring less fracture-fracture connectivity to achieve large-scale permeability structures, while the less porous South Australia tight gas formations are substantially more dependent on developing fracture-fracture connectivity to achieve large scale permeability structures.

**R3 and Fracture Connectivity**

Fig 9 illustrates the physical significance of fractures as inferred from the $R_3$ phenomenology displayed in Figs 2-8. The two figure panels display fluid flow velocity fields computed for flow from an entry wellbore to an exit wellbore. Near the wellbore the flow velocities are high (green), then grade to lower velocities away from the wellbore (yellow to red). Arrows indicate the sense of flow between the wellbores. The difference between the two plot panels is that the left panel is the velocity field of a low value of $\alpha$ and the right panel is that for a high value of $\alpha$. Fluid flow in poroperm media conforming to empirical rules $R_1$ and $R_2$ as a function of parameter $\alpha$ in $R_3$ is thus demonstrated to be a matter of realizing a range of fracture-connectivity and permeability magnitudes for a given porosity distribution. That is, $R_3$ is equivalent to an empirical statement that flow connectivity within a population of grain-scale cement-defects permits and promotes large-scale percolation pathways through the long-range spatially-correlated connectivity of grain-scale defects implied by $R_1$.

**Discussion/Conclusion**

Natural range of fracture-connectivity parameter $\alpha$ and prospects for controlled stimulation of permeability in EGS volumes

The $R_3$ phenomenology of Figs 2-8 illustrate a general tendency for in situ poroperm data to exhibit a range of $\alpha$ values – some permeability distributions are near-normal with lower overall permeability (i.e., have lower $\alpha$) and some are ‘long-tailed’ or ‘lognormal’ with overall higher permeability (i.e., have higher $\alpha$). The simple progression of permeability distributions pictured in Fig 2 for a fixed porosity in an otherwise unvarying medium is paralleled for in situ data only in loose general trend. Many other geological variables are in play for ensembles of in situ poroperm data. Nonetheless, the existence of a general tendency of in situ poroperm data to show a range of $\alpha$ can be interpreted to mean that, even amidst the complications of in situ geology, we have evidence that spatially varying fracture-connectivity is a feature of in situ flow and reservoir structure outside the accidents of geology. This inference is explicitly supported by the flow structure computations of Fig 9. We are thus invited to consider that where natural processes can generate a range of fracture-connectivities, we might be able to achieve a parallel result by intervening to inject grain-scale fracture damage that would naturally lead to increased fracture connectivity, and hence increased permeability, of the treated crustal volume.
In a companion paper (Pogacnik et al 2012) we compute the strain field of an inhomogeneous poroperm medium subjected to high wellbore pressurization, and find that we can expect such treatment to generate significant strains that could inject grain-scale damage into the medium over and above the damage already present. Given the naturally occurring range of \( \alpha \text{ in situ} \), we can expect that such treatments are at least approximately duplicating the nature processes of damage injection and enhanced fracture connectivity and permeability leading to higher values of \( \alpha \).

**Trace Elements and Ore Grade Distributions and Their Relation to Fracture-Borne Flow**

The general picture of trace element and ore grade distributions is that some distributions are normal while a large number are ‘lognormal’ (Ahrens 1954a, 1954b, 1957, 1963; Koch & Link 1971; Link & Koch 1975; Clark & Garnett 1974; Gerst 2008). Ahrens (1963) cites 32 instances of lognormal trace element distributions against two of near-normal distribution (with five instances judged uncertain). A snapshot summary of aspects of Ahrens (1963) trace element distributions in Figs 10-11 visually connects with the poroperm distribution systematics illustrated in Figs 2-8. Gerst (2008) shows numerous statistical quantile plots for the logarithm of copper ore grade for a range of ore deposit types, thus directly indicating the predilection for ore grades to be lognormally distributed, and concludes that statistical tests support normal distributions for most of the observed logarithm(ore grade) and logarithm(ore tonnage) distributions.

The simple point of the trace element abundance and ore grade distribution phenomenology is that in both cases the underlying cause could be the \( R^3 \) poroperm distributions empirics that describe \textit{in situ} fracture system heterogeneity. Crustal fracture systems that are poor in fracture connectivity lead to relatively low-abundance/ore-grade normally distributed deposition systems, while crustal fracture systems that are rich in fracture connectivity lead to relatively high-abundance/ore-grade ‘lognormally’ distributed deposition systems.

**Conclusion**

If these discussion items are accurate, we can learn from the wide-spread nature of clastic reservoir well-core poroperm, trace element abundance, and ore grade distributions that the \( R^3 \) poroperm phenomenology is a common feature of crustal rock flow systems and its properties can likely be exploited in interest of EGS permeability enhancement. We further explore this strain damage mechanics in Pogacnik et al. (2012).

**References**


