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Three-Dimensional Inversion of Magnetotelluric Data on a PC; Methodology and Applications to the Coso Geothermal Field

Virginie Maris¹, Philip Wannamaker², and Yutaka Sasaki³

¹University of Utah, Dept. of Geology and Geophysics, Salt Lake City, UT U.S.A.
²University of Utah, Energy & Geoscience Institute
³Kyushu University, Fukuoka, Japan

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ABSTRACT

We describe here efforts in technology development to invert magnetotelluric (MT) data collected in geothermal settings for three-dimensional resistivity models using desktop PC’s or small clusters. A finite difference scheme is utilized for the forward problem, with various options to compute the parameter Jacobians, and parameter step estimates are defined using an explicit Gauss-Newton step. The paper examines mesh and inversion parameter design suitable for MT data collected on the east flank of the Coso geothermal field by using synthetic models based on existing ensembles of two-dimensional inversions. The algorithm design can include static shifts at each MT site as additional parameters solved in the inversion. Initial inversion results for the Coso data set qualitatively resemble previous models from 2-D inversion stitches and from massively parallel 3-D inversion, although improvements are still sought in fit and convergence. At present, run times on modern serial desktops are 2-3 days, with the great majority of time being spent on the parameter Jacobians using reciprocity. An important developmental goal is to speed Jacobian calculation through alternate means such as a good approximation using integral equations.

Introduction

Imaging and interpreting subsurface electrical resistivity structure can assist in understanding complex geothermal systems. In addition to imaging subsurface structures, electrical resistivity measurements can detect variations related to fluid flow in geothermal systems, such as increased electrical resistivity contrasts resulting from increased fluid content due to fracturing, and from the development of more conductive alteration minerals as a result of flow. The magnetotelluric (MT) method, an electromagnetic (EM) geophysical technique, has been successfully used to image subsurface electrical resistivity in complex geothermal systems. Interpretation of MT data in 3-D has typically required large computing resources and long run times (Newman et al., 2005a). Moreover, the process is complicated by small-scale near-surface inhomogeneities resulting in static shifts, which are frequency-independent, site-specific scalings of the measured apparent resistivities. Here we develop and...
utilize the 3-D MT inversion method of Sasaki (2004), which uses a Gauss-Newton inversion approach to solve for static shift and subsurface conductivity distribution parameters simultaneously, and can handle moderate-sized MT data sets on a personal computer.

One of our goals is to apply the developed algorithm to MT data collected at the East Flank of the Coso geothermal area, a high-temperature power-producing field in southeastern California (Monastero et al., 2005). Just over 100 sites were collected in this difficult environment by Wannamaker et al. (2004) as part of the U.S. Dept of Energy’s Enhanced Geothermal Program research (Sheridan et al., 2003) (Figure 1). Interpretation of these data has included 2-D stitched vertical slices (Wannamaker, 2004; Newman et al., 2005b), and 3-D inversion using a massively parallel computer (Newman et al., 2005a). An important structure appearing in these interpretations is a high-angle conductor most prominent in the southwest East Flank sector correlated with its producing reservoir. The previously published inversion results however do not address the removal of static shift, instead opting to reproduce them with fine discretizations, and the 3-D efforts have taken weeks of parallel computer run time. Our longer term goal is to carry out 3-D inversion of the Coso MT data using an improved and efficient PC-based algorithm, solving for subsurface conductivity structure and static shift parameters simultaneously. Work to date reported here includes tests on synthetic data to ensure that the inversion parameters have been correctly chosen and that the conductivity structure is recoverable, with an initial inversion attempt of the Coso data themselves.

Inversion Scheme

The MT method consists of measuring the variations in the magnetic field surrounding the earth, originating from the magnetosphere and thunderstorm activity, and the resulting electrical currents induced in the earth (Vozoff, 1991). The ratio of horizontal electric to magnetic fields forms the impedance, with the real part referred to as the apparent resistivity and the imaginary part as the phase. This ratio is dependent on frequency, source field polarization, and subsurface geoelectric structure. Small-scale, near-surface inhomogeneities result in a frequency-independent vertical shift of the apparent resistivity, which, when expressed in natural log units, can be written as

\[
\ln \rho_a = \ln \rho_s + s
\]

where \( \rho_a \) is the apparent resistivity, \( \rho_s \) is the subsurface resistivity, and \( s \) is the size of which varies with site and source polarization. The phase remains unaffected.

In our inversion approach (Sasaki, 2004), the forward problem can then be expressed as:

\[
d_{\text{pre}} = \mathbf{A}(\mathbf{m}) + \mathbf{G}s
\]

where \( d_{\text{pre}} \) is a vector of predicted data, composed of the natural logarithms of apparent resistivities and phases; \( \mathbf{A} \) is a non-linear forward operator, based on Maxwell’s equations; \( \mathbf{m} \) is a vector of subsurface conductivity model parameters; \( s \) is a vector of static shift parameters; and, \( \mathbf{G} \) is a matrix of ones and zeros, used to relate corresponding predicted apparent resistivity values and static shifts. Applying the differential form of Maxwell’s equations over a staggered finite difference grid, the secondary fields and predicted MT responses are solved using preconditioned conjugate gradient (BCG) relaxation scheme with divergence correction.

To solve the inverse problem for both \( \mathbf{m} \) and \( s \), and to constrain the solution, the following objective function is defined:

\[
\psi = \| \mathbf{W}(\mathbf{d}_{\text{pre}} - \mathbf{d}_{\text{obs}}) \|_2 + \lambda^2 (\| \mathbf{Cm} \| + \alpha^2 \| \mathbf{m} - \mathbf{m}_0 \|_2^2) + \beta^2 \| s \|_2^2
\]

(2)

with \( \| \cdot \|_2 \) indicating the L2 norm. The first term \( \| \mathbf{W}(\mathbf{d}_{\text{pre}} - \mathbf{d}_{\text{obs}}) \|_2 \) represents a measure of the misfit between the observed data, \( \mathbf{d}_{\text{obs}} \) and the predicted data, weighted by the reciprocal of the data standard deviation which are held in diagonal matrix \( \mathbf{W} \). The term \( \| \mathbf{Cm} \| \) imposes a smoothness constraint, with the roughness of the subsurface conductivity structure defined by the Laplacian operator \( \mathbf{C} \). The term \( \| \mathbf{m} - \mathbf{m}_0 \|_2 \) imposes an adherence constraint, limiting how much the new model can differ from the a-priori model, \( \mathbf{m}_0 \). The term \( \| s \|_2 \) acts as a constraint on the size of static shift, and assumes that the static shift parameters are Gaussian distributed with a mean of zero.

Minimization of the functional (2) yields the following system of equations for the direct Gauss-Newton (G-N) parameter estimator:

\[
\begin{bmatrix}
\mathbf{W} \mathbf{F}_n \mathbf{W} \\
\lambda \mathbf{C} \\
\lambda \alpha \mathbf{I} \\
0
\end{bmatrix}
\begin{bmatrix}
\mathbf{m}_0 \\
\mathbf{s}_0 \\
\mathbf{m}_0 \\
0
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{W}(\mathbf{F}_n \mathbf{m}_0 + A(m_0) - \mathbf{d}_{\text{obs}}) \\
0 \\
\lambda \alpha \mathbf{m}_0 \\
0
\end{bmatrix}
\]

(3)

where \( \mathbf{m}_0 \) is the starting model, the sensitivity \( \mathbf{F}_n \) is the matrix of derivatives (Jacobians) of \( \mathbf{A} \) with respect to the resistivity model parameters, and \( \mathbf{I} \) is the identity matrix. This system is solved using the modified Gram-Schmidt method. At this time, we retain a direct G-N approach because of experience with its good convergence in a small number of iterations (6-10 typically).

With direct G-N solutions like this, the most time consuming portion for all but very large models is computing the entries of \( \mathbf{F}_n \). In a standard approach utilized by other direct codes (e.g., Siripunvaraporn et al., 2005), through the concept of reciprocity one requires auxiliary forward solutions with several finite source orientations at each MT data receiver in the model to obtain both electric and magnetic field sensitivities. For 1-D starting models, these solutions are just various Hankel transforms requiring modest computer time (e.g., Wannamaker et al., 1984). For arbitrary subsurface structure as would form in subsequent inversion iterations, we are faced with full BCG solutions for each source, or potentially hundreds of times the computer effort of the initial MT forward solution. Speeding this step is a prime goal of our research for which we will seek rapid but good approximate integral equations solutions (Wannamaker and Sasaki, 2003).
The data misfit is calculated as

\[ \text{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} W_i \left( \ln \left( \frac{\rho_{\text{obs}}}{\rho_{\text{pr}}} \right) \right)^2} \]

where \( N \) is the number of data. As the predicted data approaches the observed data to within the data standard deviation, the misfit should approach 1.

### Tests on Synthetic Data

Prior to inverting the measured Coso MT data, tests were carried out on synthetic data to evaluate the inversion parameters and whether the anticipated target would be recoverable. The model used to generate the synthetic data was based on the results obtained by 2-D stitched inversion of E-W subprofiles (Newman et al., 2005b). It includes a layered background, shown in Table 1, derived from the integrated TM mode impedance of the dense array MT measurements spanning the east-west extent of the Coso east flank (Figure 1) (Wannamaker, 2004; Newman et al., 2005b). Emplaced in this 1-D host is a 10 \( \Omega \cdot \text{m} \), 2.5 km thick brick at 500 m depth, extending 3 km north to south, and 1 km west to east. For testing, and to limit run times for early evaluation, 56 sites and seven frequencies ranging from 0.6 to 230 Hz were chosen, emulating a subset of available Coso data (Figure 2). Arbitrary error envelopes of 1.1 \( \Omega \cdot \text{m} \) apparent resistivity and 2.6° phase were assigned to the data. MT stations are assumed to be on a horizontal plane. The finite-difference mesh consists of 62 nodes in the x (easting) direction, 56 nodes in the y (northing) direction, and 37 nodes vertically of which 10 are above the air-earth interface. The inversion domain consists of 17x14x15 blocks in x, y and z directions, which is more coarsely discretized than that used for the Coso data set. The half-space resistivity of the starting model was calculated by the program, based on the input apparent resistivity data; no explicit information regarding the layered background was input to the inversion.

Preliminary inversion results are shown in Figure 2. The model obtained has a misfit of 1.3, after four model parameter updates (Figure 3). The recovered quasi 1-D background resistivity is generally consistent with, and slightly more resistive than the background used in generating the synthetic data (Table 1). Within the resistive background layers, a region of decreased resistivity occurs, coincident with the conductive brick target, but diffuse and of lower contrast also.

### Table 1. Thickness and resistivity of layers in background model derived from 1-D inversion of integrated TM impedance of dense MT array line, and recovered from inversion of the test data. Table note that the vertical discretization used in inversion of the synthetic data and Coso data was based on the thickness derived from the dense MT array line.

<table>
<thead>
<tr>
<th>Thickness (km)</th>
<th>Resistivity (( \Omega \cdot \text{m} ))</th>
<th>Resistivity (( \Omega \cdot \text{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derived from 1-D inversion of integrated TM impedance of dense MT array line</td>
<td>Recovered from inversion of synthetic data</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>24.7</td>
<td>10.6</td>
</tr>
<tr>
<td>0.063</td>
<td>5.7</td>
<td>10.3</td>
</tr>
<tr>
<td>0.079</td>
<td>8.5</td>
<td>8.3</td>
</tr>
<tr>
<td>0.100</td>
<td>15.4</td>
<td>12.6</td>
</tr>
<tr>
<td>0.126</td>
<td>43.2</td>
<td>28.4</td>
</tr>
<tr>
<td>0.158</td>
<td>69.8</td>
<td>64.3</td>
</tr>
<tr>
<td>0.199</td>
<td>89.5</td>
<td>117.3</td>
</tr>
<tr>
<td>0.251</td>
<td>107.9</td>
<td>160.4</td>
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<tr>
<td>0.316</td>
<td>124.8</td>
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<tr>
<td>0.397</td>
<td>137.8</td>
<td>143.9</td>
</tr>
<tr>
<td>0.500</td>
<td>144.1</td>
<td>108.5</td>
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<tr>
<td>0.630</td>
<td>140.2</td>
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<tr>
<td>0.792</td>
<td>122.0</td>
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<tr>
<td>0.998</td>
<td>83.9</td>
<td>44.9</td>
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<tr>
<td>1.256</td>
<td>7.5</td>
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<tr>
<td>1.581</td>
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<tr>
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<td>16.2</td>
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<tr>
<td>8.972</td>
<td>7.1</td>
<td>16.2</td>
</tr>
</tbody>
</table>
Inversion of Coso MT Data

The Coso MT data included in the inversion consists of the apparent resistivity and phase measurements of the off-diagonal impedance elements (xy and yx modes), measured at 14 frequencies ranging from 0.3 Hz to 230 Hz, for 98 of the 102 stations in the area of interest. The x-axis is true north, and y is east. Pseudo-sections of the observed data from the diagonal profile (Figure 1) are shown in Figure 4, for frequencies of 1 to 100 Hz.

The finite-difference mesh consists of 62 x 56 x 37 nodes in the x, y, and z directions, and the inversion domain of 27 x 25 x 15 blocks. These dimensions were selected to ensure that each inversion block contains two finite-difference cells in both the x and y directions, and to maintain, as much as possible, an empty inversion block between adjacent stations. Where necessary, station positions were shifted so as to fall on nodes of the finite-difference mesh at the center of each inversion block. The starting model at iteration 1 consists of a halfspace, the resistivity of which was calculated by the program based on averaging the apparent resistivity data. Four iterations were completed in approximately three days, with each iteration requiring rigorous sensitivity calculation of a 3-D conductivity model taking nearly a day.

Pseudo-sections of the predicted data after four iterations are shown in the bottom and middle panels of Figure 4; the misfit at each iteration on Figure 2. The model obtained, shown in Figure 5, has a misfit of 10.7. A large resistive feature towards the center of the domain is flanked to the east by a north-south trending, less than 1.5 km thick conductor, which we correlate with deepening sediments of the Coso Wash graben. The resistive horst also is flanked on the opposite side by a thicker, south-trailing conductor which we tentatively correlate with the conductor below the southeast flank discussed previously, although it currently appears overly smoothed. The resistivities obtained from our inversion have a smaller range than those from the dense array MT data, but exhibit a qualitatively similar pattern of conductivity distribution as that obtained by 2-D stitched inversion of E-W subprofiles (Newman et al., 2005b). Although a satisfactorily low misfit has not been achieved yet, these preliminary results are encouraging. Further refinement of the model is anticipated by improving the choice of regularization parameters, by including the layered background as a-priori model in the inversion, and by increasing the number of iterations.

Figure 3. Plot of the misfit versus iteration for the test model and for the Coso MT data inversion results.

Figure 4. Apparent resistivity pseudo-sections of the diagonal profile shown in Figure 1. Stations adjacent to but not falling on the profile have been projected onto it, with distance relative to MT station 95. Observed data are on the top panels; the second row of panels consists of the predicted data calculated from the recovered model parameters, the static shift and the inversion block resistivities combined. Bottom panels show the contribution to the total response of the subsurface conductivity structure separately from that of the applied static shift.
Conclusions and Plans

The prototype 3-D inversion algorithm with which we have worked to date appears viable for moderate sized data sets such as collected at Coso; initial tests look promising. It seems likely that the full 100+ stations of the survey will be fittable in a modern desktop with inversion runtimes on the order of 2-3 days. However, marked improvements to the code are possible and will be pursued. To speed calculating the sensitivity matrix, we plan to include an integral equations based formulation as an option to replace the finite-difference sensitivity. Additional important improvements in Jacobian accuracy will be obtained using a full layered host for the finite source reciprocal calculations. Use of a Marquardt factor to lend a component of steepest-descent convergence will be studied. Storage will be saved through use of a depth-expanding parameter grid such as we have invoked in 2-D and through allocatable arrays. Finally, we plan to develop a parallelized version for porting to relatively inexpensive, Beowulf-type clusters such as we have recently obtained.

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