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An approximate analytical solution for the temperature distribution in a slab caused by the flow of water up a fault is obtained by assuming that the heat lost from the water is proportional to the temperature difference between the water and the slab at a distance far from the fault. Comparison of the analytical solution with the numerical solution of Sorey (1975) shows generally good agreement except in the region very close to where the fluid exits from the fault. The disagreement is caused by the approximate boundary condition used in the analytical solution not being valid in that region. A temperature profile in well RRGE-1 at the Raft River geothermal area, Idaho is compared to temperature distributions calculated at various distances from the fault. The gross features of the measured and calculated profiles agree if the well is assumed to be near the fault and the flow up the fault is about 6 liter/sec per kilometer of fault length.

INTRODUCTION

The temperature distribution caused by the flow of hot water up a narrow fault is important to the interpretation of subsurface temperature patterns in geothermal systems. For the case of no boiling, Sorey (1975), Blackwell and Chapman (1977), and Kilty, Chapman and Mase (1978) have obtained numerical solutions. Lowell (1975) has obtained an approximate analytical solution for the temperature distribution in the fault. The purpose of this report is to obtain an approximate analytical solution for the steady-state temperature distribution in the fault and surrounding ground and compare results with temperatures measured in a drilled well at the Raft River geothermal area, Idaho.

MATHEMATICAL SOLUTION

The physical situation and coordinate system are shown in figure 1. The problem will only be solved for one half the domain, so one half of the volume flow per unit strike length Q enters the bottom of the fault at temperature \( T_1 \) and flows up to the surface. The upper and lower boundaries are fixed at \( T_0 \) and \( T_1 \) respectively. The origin of the coordinate system is taken at the lower left-hand corner. The differential equation and boundary conditions are:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (1)
\]

\[
\frac{\rho_c Q}{2} \frac{\partial T}{\partial y} \bigg|_{x=0} = k \frac{\partial T}{\partial x} \bigg|_{x=0} \quad (2)
\]

\[
T(x,0) = T_1 \quad (3)
\]

\[
T(x,D) = T_0 \quad (4)
\]

where \( \rho_c \) and \( c_w \) are the density and specific heat of water and \( k \) is the thermal conductivity of the material in the slab. The solution to this problem is difficult because the eigenfunctions obtained from the partial differential equation (1) are not compatible with eigenfunctions needed to satisfy equation (2).

As suggested by Lowell (1975), we can replace (2) by the boundary condition

\[
\frac{\rho_c Q}{2} \frac{\partial T}{\partial y} \bigg|_{x=0} = -\frac{k}{R} (T \bigg|_{x=0} - T(\omega,y)) \quad (5a)
\]

\[
T(\omega,y) = T_1 + (T_0 - T_1) y/D. \quad (5b)
\]
The physical interpretation of this new boundary condition is that the energy lost by the water flowing in the fault, given by the left-hand side of (5a), is proportional to the temperature difference between the water in the fault and the slab at a distance far away from the fault. The constant of proportionality \( R \) is the distance to which the flow in the fault has an appreciable effect on the temperature distribution in the slab. The advantage of equations (5) is that the temperature distribution in the fault may be solved first, and then applied as a boundary condition to equations (1), (3), and (4).

Defining dimensionless variables

\[
y^* = \frac{y}{D} \\
x^* = \frac{x}{D} \\
T^* = \frac{T - T_1}{T_1 - T_0} \\
Q^* = \frac{\rho c_w Q}{2k} \\
R^* = \frac{R}{D},
\]

we can integrate equations (5) to obtain

\[
T^*(0, y^*) = -y^* + Q^* R^* \left( 1 - e^{-y^*/Q^* R^*} \right).
\]

The solution to equation (1), with boundary conditions (3), (4), and (7) may be obtained by the method described in Carslaw and Jaeger (1959, sec. 5.2) and may be written as

\[
T^* = -y^* + \sum_{n=1}^{\infty} A_n \exp(-n \pi y^*) \sin(n \pi y^*)
\]

\[
A_n = \frac{20 n \pi}{\pi^2} \left\{ \frac{1 - \cos n \pi + e^{-1/Q^* R^*}}{1 + (Q^* R^* n)^2} \right\}.
\]

The solution is strongly convergent everywhere but near \( x^* = 0 \). (At \( x^* = 0 \), the temperature is given by (7)).

In order to complete the solution, the adequacy of the approximate boundary condition (5) must be checked and a value for \( R^* \) obtained. This evaluation can be made using the numerical solution of Sorey (1975). His mathematical problem is slightly different, because the fault is modeled as a conduit of finite width. The conduit width is a small fraction of the fault depth, so the solution should not be strongly affected. Sorey (1975, p. 48a) presents a plot of temperature of the water as it exits the fault versus flow. The plot is the same form as equation (7), and the value of \( R^* \) needed to match his calculations is 0.15. He also presents isotherms in the slab for \( Q^* = 3.2 \). Isotherms calculated from equations (8) for \( R^* = 0.15 \) are very different from his calculations. The value of \( R^* \) can be adjusted to maximize the agreement between the results of equations (8) and his calculations. A value of \( R^* \) of 0.37 produces the isotherms shown in figure 2. The agreement between the two sets of results is very good except near the upper left-hand corner. The reason for the disagreement is that the approximation involved in using equations (5) breaks down in that corner. The constant-temperature upper boundary causes large values of horizontal temperature gradient near where the flow exits the fault, so equations (5) are not an accurate representation of the true boundary condition (2). This interpretation is confirmed when equations (2) and (5) are combined with the solution (8) to calculate \( R^* \) as a function of depth near the fault (\( x^* \approx 0.02 \)). From \( y^* = 0 \) to around 0.8, \( R^* \) is reasonably constant. From \( y^* = 0.8 \) to 1.0, the value of \( R^* \) is a strong function of \( y^* \), and it gets quite small near \( y^* = 1.0 \). Thus, the solution using the approximate boundary condition is valid except in the upper left-hand corner of the domain. To use the analytical solution, the outflow temperature \( T^* (0, 1) \) as the water exits the fault can be evaluated from equation (7) using \( R^* = 0.15 \). The temperatures in the slab are then calculated from equations (7) and (8) with the same \( Q^* \) but now \( R^* \) takes the value of 0.37. The temperatures are valid except when \( y^* \) is greater than 0.8 and \( x^* \) is less than 0.1.

Figure 2. Comparison of isotherms obtained from equations (7) and (8) for \( R^* = 0.37 \) (broken line) with those obtained from a numerical solution by Sorey (1975). \( Q^* = 3.18 \).
APPLICATION TO A WELL AT RAFT RIVER, IDAHO

Well RRGE-1 was drilled from January 4, 1975 to March 31, 1975 to a total depth of 1521 m. The well was flow tested in April, 1975. Temperatures measured on October 16, 1975 to a depth of 1085 m (approximate depth of casing) are shown on figure 3. Measurements were made through a packing gland to contain the overpressure of approximately 10 bar gauge at the wellhead. Temperatures were measured using a glass-enclosed bead thermistor (25,000 ohms at 25°C) attached to a four-conductor cable. Resistances and depths were sampled at intervals of 5 ft (1.5 m). During drilling, a zone of lost circulation was encountered at a depth of approximately 460 m and about 5 million liters of water was taken by the well until the casing was installed. The reversal centered around 450 m is the remaining perturbation from this large fluid loss. Additional nonequilibrium is indicated by a temperature near the surface of the ground significantly above the mean annual temperature of 10 to 11°C found in shallow drillholes.

Figure 3. Temperature log obtained in well RRGE-1 (16 October 1975) Raft River geothermal area, Idaho. Wellhead pressure 10 bars gauge, no flow. Cased depth (C.D.) and total depth (T.D.) shown.

Figure 4. Temperatures versus depth at several distances x from fault (D is fault length) for Q*=0 and R*=0.37.
The well was drilled to intersect the Bridge fault at depth and is interpreted to have done so between 1240 and 1300 m (Williams et al., 1976). Flowing enthalpy indicates that water of 147°C feeds the well (Williams et al., 1976). The geometry of the geothermal system is more complex than the geometry in figure 1; the fault is not vertical but dipping and at shallow levels there is probably leakage into cold-water aquifers. The Schmitt hot well is located about 0.8 km away from RRGE-1 along the strike of the fault at depth and is interpreted to have done so. Well RRGE-2 is located about 1 km away from RRGE-1 along the strike of the fault. It has a similar profile to RRGE-1 though with less curvature. Well RRGE-2 is about 2 km from the line joining the locations of RRGE-1 and 2 and shows a temperature reversal. This reversal indicates that flow patterns are significantly more complicated than the simple geometry of flow up a fault and then out of the ground. Nevertheless, the calculations for the simple geometry are suggestive as to some of the behavior of temperatures in the Raft River geothermal area. It is worth noting that there are alternative explanations for the temperature distribution of RRGE-1. Bredehoeft and Papadopulos (1965) have presented a solution for the temperature distribution caused by uniform upflow in a porous medium. The temperature-depth profile of RRGE-1 could be matched by their type curves to yield an equivalent Darcy seepage velocity. Another way to produce curvature in a vertical temperature profile is with a horizontal flow of water that is losing or gaining energy in the direction of flow.

CONCLUSION

The idealized physical problem considered here is unlikely ever really to occur; however, its application to field problems in an approximate sense is very useful. One need only know the exit temperature and source temperature for flow up a fault in order to determine the rate of flow up the fault (Sorey, 1975). The approximate analytical solution provides useful results except in a small region near where the flow exits the fault. The application of the solution to the temperature profile measured in well RRGE-1 indicates that the well is close to the fault (geologically reasonable) and that the flow is of order 6 liter/sec per kilometer of fault length. The temperature pattern measured in RRGE-1 does not agree in detail with the calculated profile, however, this reflects complexities ignored in the simple problem considered here.

REFERENCES


